

About negative refraction and left handed materials

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Abstract

Here I present an overview of recent studies of the phenomenon of negative refraction and left-handed materials. I will discuss some basic questions pertinent to the problem. It is pointed out that the current claims of negative refraction and left handed materials are not conclusive. To support our consideration, I will start with the fundamental physics.

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I. PRELUDE

Waves surround us. Direct human communication is mainly conveyed by acoustic waves, and is enriched by gestures which are passed into our eyes through optical waves. Indirect human conversation is transmitted via electromagnetic (EM) waves. Nowadays electronic waves are also everywhere in our daily experiences such as audio & video systems, computers, and Nintendo games. Among many interesting properties pertinent to waves, refraction is probably the most common phenomenon in our daily life.

In simple terms, refraction refers to waves bending in passing obliquely from one medium to another. It is a common observation that a beam of light bends when it enters glass or water at a tilted angle. Rainbow which must be a phenomenon known to every one who could see is another example of wave refraction. More specifically, refraction occurs when waves pass across the interface between two different effectively uniform media. Briefly speaking, the effectiveness refers to that inside the medium a phase vector (\vec{k}) can be effectively defined for a given frequency. In an isotropic medium, for example, a plane wave can be supported and is described in the form like $A \exp(i\vec{k} \cdot \vec{r} - i\omega t)$. Therefore to study the phenomenon of refraction, it is important to verify whether the media are effective or not.

In the world with which we are so far most familiar, EM waves or lights propagate according to the rule of right hand. That is, the electric and magnetic vector fields \vec{E} and \vec{H} , and the phase vector \vec{k} form a righthanded coordinate system. In other words, the direction \vec{k} is along the direction of $\vec{E} \times \vec{H}$. Materials in which \vec{k} and $\vec{E} \times \vec{H}$ are in the same direction are called isotropic media. The direction of $\vec{E} \times \vec{H}$ is called the ray or beam direction.

In nature, there is another class of materials. In these materials, although the direction of \vec{k} may not be exactly along the direction of $\vec{E} \times \vec{H}$, the angle between the two is smaller than 90 degree, in the mathematic terms, $\vec{k} \cdot (\vec{E} \times \vec{H}) > 0$. This class of materials is called optically anisotropic media. Calcite crystals and YVO₄ bi-crystals are two examples of anisotropic media.

Wave refraction can appear dramatically different when crossing the interface between either isotropic or anisotropic media. For example, when an optical wave crosses an interface between two isotropic media, the refracted wave will always bend towards the other side of the incidence with regard to the normal of the interface. The refraction and incident angles

obey Snell's law[1]. When the wave passes an interface between an isotropic medium and an anisotropic medium, however, a phenomenon called amphoteric refraction can occur. That is, the refracted ray may bend either towards the same side as the incidence or the other side of the incidence, depending on the incident angle and the orientation of the anisotropic medium. In this case, Snell's law fails[2]. For a uniaxial crystal, for instance, there are two eigen modes of plane waves: the ordinary and extra-ordinary modes[2]. The dispersion relation of the ordinary mode is isotropic and this mode of light behaves as the light beam in an ordinary material, while the equal frequency surface or normal surface[2] of the extra-ordinary mode is ellipsoidal with the optic axis being the axis of revolution. The allowed wave vectors are those vectors pointing from the center of the ellipsoid to its surface. For this mode, the energy velocity is shown to be equal to the group velocity determined by $\vec{v}_g = \nabla_{\vec{k}}\omega(\vec{k})$ [2].

Figure 1 is an illustration of refraction across a boundary between two isotropic media whose refraction indices are denoted as n_1 and n_2 respectively. In this case, the ray direction and the phase vector's direction are the same. The boundary condition $\vec{k}_1 \cdot \vec{r}_{\parallel} = \vec{k}_2 \cdot \vec{r}_{\parallel}$ leads to Snell's law

$$n_1 \sin \theta_i = n_2 \sin \theta_r, \quad (1)$$

where θ_i and θ_r denote the incidence and refraction angles respectively.

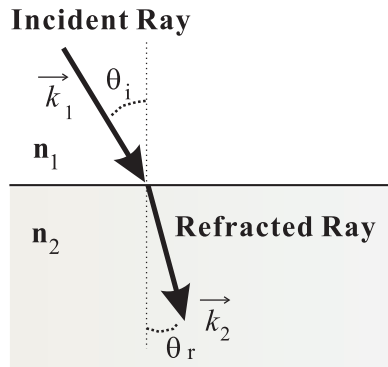


FIG. 1: Illustration of refraction at the interface between two isotropic media.

Figure 2 depicts the amphoteric refraction of the extraordinary light at a flat interface between an isotropic medium and an anisotropic medium. The boundary condition requires that the phase vectors \vec{k} be continuous, when projected onto the interface. The ray direction, denoted by the time averaged Poynting vector $\vec{S} = \frac{1}{2}\text{Re}[\vec{E} \times \vec{H}^*]$, is normal to the equal

frequency surface. Here the refracted ray is bent towards the same side as the incident ray in (a) while the ray is tilted to the other side in (b). Such an amphoteric refraction phenomenon can be readily observed at the interface between air and calcite crystals[3]. In this case, Snell's law in Eq. (1) fails. A modified version of the angle dependence should be introduced[3].

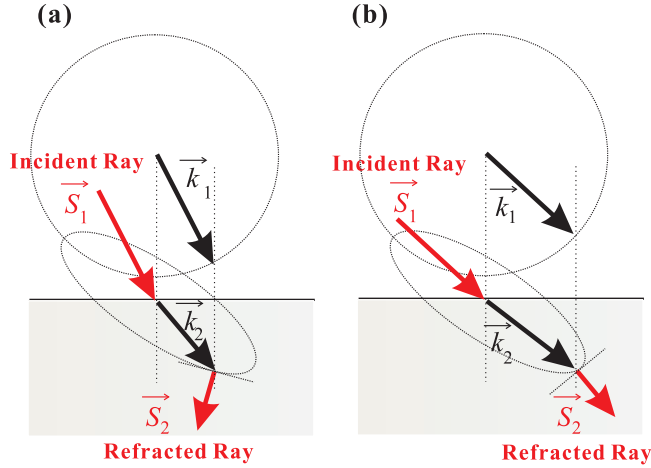


FIG. 2: Illustration of amphoteric refraction at the interface between an isotropic medium and an anisotropic medium: (a) negatively appearing refraction; (b) positive refraction

II. INTRODUCTION

Equipped with the above textbook knowledge, we are ready to discuss the much debated issue of Left Handed Materials (LHMs) and the associated phenomenon of negative refraction (NR).

The concept of LHMs was introduced by Veselago many years ago [4]. What is Left Handed Material? According to Veselago [4], a left handed material refers to electrodynamics of substance with simultaneously negative values of permittivity ϵ and permeability μ . An immediate deduction from this perception is that the material should have a negative refraction index, as $n = \sqrt{\epsilon}\sqrt{\mu} = -\sqrt{|\epsilon\mu|}$. Another important outcome is that the energy ray \vec{S} and the phase vector \vec{k} will no longer point to the same direction. Instead, they will be opposite to each other in an isotropic LHM or make an angle bigger than 90 degree in an anisotropic LHM, i. e. $\vec{k} \cdot \vec{S} < 0$.

A negative refractive index would lead to a few peculiar phenomena. Taking the propagation of EM waves from a normal material through an isotropic LHM as the example, some of the expectations are described in Fig. 3. Inside the LHM, the phase vector points (Black arrows) to the opposite direction of the energy rays, denoted by the red arrows. Fig. 3(a) illustrates the negative refraction. Here, Snell's law applies as

$$n_1 \sin \theta_i = (-|n_2|)(-\sin \theta_r). \quad (2)$$

The second prediction is the imaging by a slab of LHM as shown in Fig. 3(b). For a slab of normal refractive materials, isotropic or anisotropic, no image can be made across the slab. There will be no image inside the slab either. Here, Fig. 3(b) shows that two images can be made across the slab of LHM when a wave is transmitted from a point source.

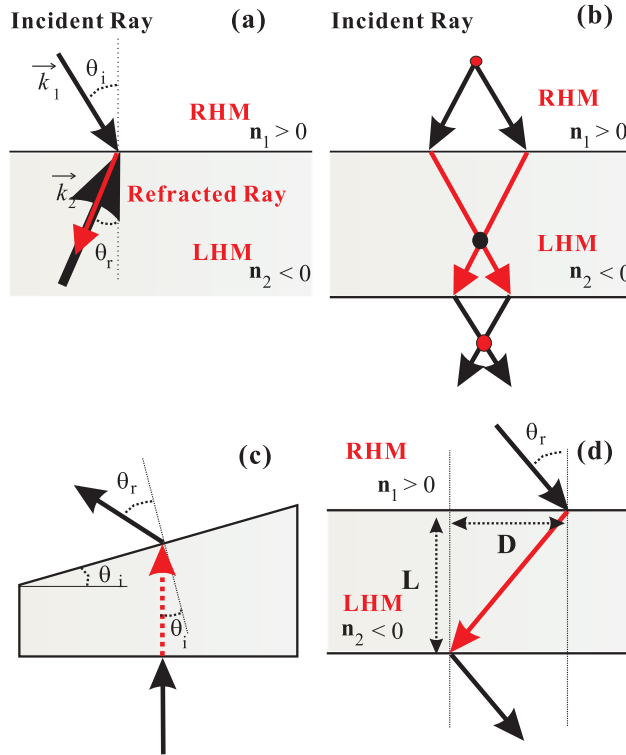


FIG. 3: Illustration of a few expected phenomena for EM wave propagation through a LHM: (a) negative refraction; (b) Imaging by a flat slab of LHM; (c) negative refraction by a prism of made of LHM; (d) negatively shifted outgoing waves. Here the red arrows denote the energy ray inside the LHM. Here we have ignored the high order internal reflections.

The third possibility is the negative refraction by a prism of LHMs. This is described

in Fig. 3(c). A plane wave is incident to the horizontal side of the prism. At the long side, the incident angle is determined as the prism angle. From the refraction angle θ_r , one can deduced the refractive index using Snell's law. If it is tilted towards the left side, the refractive index is negative. Otherwise, the refractive index is positive.

When a plane wave is incident on to a flat slab of LHMs, the outgoing wave will be shifted towards the left side, as shown in Fig. 3(d). The negative refractive index can thus be deduced as

$$n_2 = -\frac{n_1 \sin \theta_i}{\sin(\tan^{-1}(D/L))}. \quad (3)$$

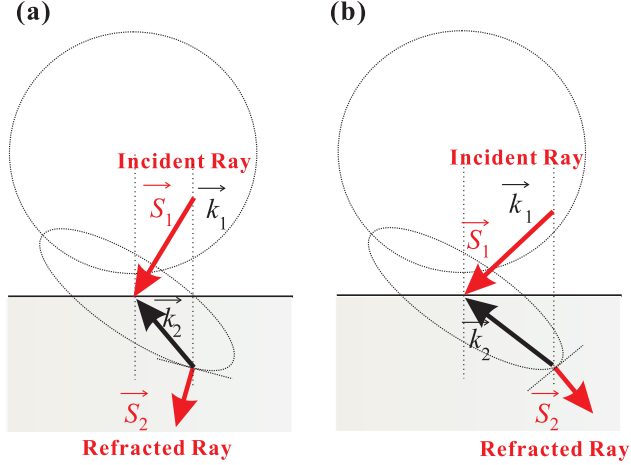


FIG. 4: Illustration of amphoteric refraction of the extra-ordinary waves at the interface between an isotropic normal medium and an anisotropic uniaxial LHM medium: (a) positive refraction; (b) negative refraction. The black and red arrows denote the phase vectors and energy rays respectively. In isotropic normal media, the energy ray follows the direction of the phase vector.

The four deductions described in Fig. 3 have actually formed the base line in the current searching for LHMs and negative refraction. A number of notes must be made.

1. An important component underlying these scenarios is that the medium can be regarded as an effective medium, in which a refractive index can be defined, and all the wave propagation will be fully describable by the Maxwell equations based upon this index.
2. Another essential point is that the LHM is isotropic, to warrant the four deductions. For an anisotropic LHM material, the amphoteric refraction can also appear, as shown

in Fig. 4. In this case, the only way to differentiate the amphoteric refraction by a normal anisotropic medium from the amphoteric refraction by an anisotropic LHM medium is to look at the phase information. In addition, Snell's law fails for the anisotropic case; therefore the refractive index cannot be readily obtained in this case.

3. The negative refraction of LHMs in Fig. 3 must be differentiated from the negatively appearing refraction due to anisotropy in Fig. 2(a). The mechanisms behind the two similar phenomena are completely different. As shown, the phase information in the two scenarios is totally different. The negative refraction of LHMs would be revolutionary not only with respect to technology, but also to fundamental physics such as quantum electrodynamic processes. To avoid confusion, we call the negatively appearing refraction due to anisotropy or any non-LHM related effect **quasi-negative refraction**, while reserving the term **negative refraction** for LHMs. Negative refraction without negative index belongs to the former category. The two categories are completely different in the mechanism.

The proposal of LHMs by Veselago did not attract much attention until 2000 when Pendry further pointed out that negative refraction makes a perfect lens[5]. Ever since then, the research on such a perfect lens and LHM has been skyrocketing in the midst of much debate. A great body of literature has been and continues to be generated.

In spite of a few challenges with regard to the concept of LHM or relevant negative refraction effects [6, 7, 8, 9], the mainstream consensus has been that some indications of negative refraction effects are affirmative[10]: positively negative. Upon inspection, however, we believe that previous evidence for negative refraction effects may not be sufficient. The negatively appearing refraction may not be caused by negative refraction. In fact, some of the phenomena perhaps should not have been considered in the context of refraction. Instead, they may be better characterized such as diffraction, deflection or the anisotropic scattering phenomenon. In the present paper, we wish to give a detailed elaboration of this point of view. Some of present results can be referred to our unpublished preprints available from the internet.

III. A BRIEF REVIEW OF THE RESEARCH ON NEGATIVE REFRACTION

There are two approaches towards the issue of negative refraction and LHMs. One is to explore all possible properties or applications of LHMs. This approach is meaningful if and only if LHMs indeed exist. The second approach, more difficult but important and fundamental, is to search for or fabricate LHMs. In this paper, I will only consider the second approach.

So far, there is only one experimental report about naturally existing materials for which the negatively appearing refraction has been vaguely related to the negative refraction[11]. As discussed later[3, 12], the negatively appearing refraction is due to the anisotropy of the materials, and is the quasi-negative refraction phenomenon as illustrated in Fig. 2. It has been pointed out above, it is important to differentiate the quasi-negative and negative refractions. Failing to recognize the difference could cause significant confusion[13, 14].

In searing for LHMs, two types of artificial materials have been proposed so far. One type involves metallic wires and split ring resonators, first made and experimentally tested by Shelby *et al.*[15], later further supported by other groups (e. g. Refs. [16, 17]). The second type is the two dimensional photonic crystals made by arrays of parallel cylinders, initiated theoretically by Notomi[18] and Luo *et al.*[19], later experimentally tested by Cubukcu, *et al.* and Parimi *et al.*[20, 21, 22]. Notwithstanding or ignoring other investigations, these explorations are the basis for later experimental or theoretical adventures in supporting the negative refraction or LHMs.

All the explorations on LHMs and negative refraction are based on at least one of the four scenarios shown in Fig. 3. For example, we can list as follows: (a) Ref. [18]; (b) Refs. [16, 19, 21]; (c) Refs. [15, 16, 17]; (d) Ref. [20]. In particular, the negative refraction index has been obtained for the resonating wire and ring systems in Refs. [15, 16, 17] and for photonic crystals in Ref. [20, 22].

IV. DISCUSSION

A. Foundation of consideration

The question we wish to discuss is whether the current evidence for LHMs and the related negative refraction is sufficient or conclusive. Moreover, if further confirmation is needed,

what could be done next. Our consideration will be based upon the following logic reasoning.

1. Point I: If a phenomenon has another possible explanation, then it is insufficient to attribute the phenomenon to a particular mechanism. In other words, if C appears to be a common phenomenon which could be due to A and B. Then C cannot be regarded as the unique consequence of either A or B alone. It is also insufficient to attribute C to either A or B. Such a degeneracy has to be verified by finer scrutiny. As mentioned earlier, measurements at various angles is to isolate isotropy effects, and the phase information is important to differentiating the quasi-negative refraction due to anisotropy and negative refraction by anisotropic LHMs. Like in quantum mechanics, the angular momentum measurement is important to differentiate different states at the same energy level when the conservation law of angular momentum holds.
2. Point II: The evidence should not have any self-conflicting point.
3. Point III: The observation should not be accidental. Further confirmation should be made to check for robustness, unless the evidence is sufficient.
4. Point IV: The theoretical support should be independent. That is, the theoretical analysis should be *ab initio*, i. e. the simulation should not use parameters from the experiment, unless there are new predictions for experiments to verify.
5. Point V: Experimental verifications with similar setups and conditions may not be considered as completely independent.

These points are important. As an illustration, let us consider the example in Fig. 2. If only measuring the energy rays (red arrows) in case (a), then applying Snell's law to the rays, one may claim that a negative refraction has been observed and a negative refractive index can be deduced. This declaration is inappropriate.

First, the claim has to rely on the following assumptions: (1) the medium is isotropic and can be described as an effective medium; so Snell's law is applicable; (2) based upon (1), it is expected that negative refraction can also be observed at other incident angles even without confirmation. Without verifying the validity of these two assumptions, one will be unable to exclude the possibility that the observation is accidental (Point III).

Second, if simply taking the measured data from Fig. 2(a) and taking the negative refractive index from applying Snell's law, one would be able to reproduce the ray direction

measured and may claim as a theoretical support. Clearly, such a theoretical support is not independent (Point IV). In addition, the support may work for one particular measurement, but fail in other extended measurements. For example, an isotropic negative refractive index would fail to explain the observation in Fig. 2(b). Now if one comes back to show that the refraction is anisotropic, then this will be in conflict with the presumption (Point II).

Third, the negatively appearing refraction in Fig. 2(a) can be explained by the medium anisotropy. Therefore it is insufficient to declare that the observation is negative refraction, with the implication of finding a LHM (Point I).

Fourth, different groups may measure the energy rays with different locations of the detector but with the same setup and condition, and then claim to reproduce each other's results. This type of supports is clearly not independent (Point V).

These considerations will guide our following discussion. We must stress again that we have no reason to criticize the experimental evidence and theoretical analysis, but we wish to consider whether they are sufficient or conclusive. If not, what can be done to improve.

B. Analysis of the evidence and methodology

1. General discussion

In all the experiments that have been performed so far, only the wave intensity field, which is proportional to the square modulus of the EM field, has been measured. There is no report of the experimental or theoretical analysis of the phase information of the systems which are claimed to be either LHMs or negatively refracting. We note that there are theoretical investigations of the phase behavior of LHMs based on the *a priori* assumption that the systems are LHMs; we will not consider this type of investigations.

Probing the refraction phenomenon and obtaining refractive index, without the phase information, will have to rely on the assumption that the medium must be effective and isotropic. So far, there is no verification on the isotropy and effectiveness. A few possible ways to check for the isotropy and effectiveness. The isotropy can be checked by rotating a sample of the medium. If a medium is effective and isotropy, the wave propagation inside the medium will not be sensitive to the fine structures of the medium, and therefore it will not be changed when the structures are uniformly disordered. So far, no experiment has

been done along this line. As pointed above, it is improper to use Snell's law to energy ray paths without confirming the isotropy and effectiveness.

2. *Negative refraction by composite materials*

The composite materials used in Ref. [15, 16, 17] are made of periodically arranged metallic wires and split ring resonators. The components form a two dimensional square crystal-lattice. The EM waves are transmitted along the direction perpendicular to the axis of the wires. The measurements have adopted the prism scenario described in Fig. 3(c). The energy rays are measured and used in Snell's law to obtain the negative refraction index.

While some questions have already raised in Ref. [7, 8], a number of other questions can be raised with this type of measurements. First, there is no verification for the isotropy of the media, which could be readily verified by the flat slab transmission scenario in Fig. 3(d). Due to their periodic structured, the materials are unlikely isotropic. If so, it would not have been proper to invoke Snell's law for the energy rays (Point II). Second, there is no information about either the wave propagation inside the media nor about the phase. One possible solution is to correlate the phase of the outgoing waves with that of the incident waves. Third, since the composite materials are made of regularly arranged split ring resonators, possible band structure effects need to be considered. The essence is to show that the structured materials can be indeed regarded as an effective medium in which a plane wave can be established. This question has already been raised by Pokrovsky and Efros[7, 8]. In addition, the interpretation of the observation in Ref. [15] has also been questioned for possible effects due to absorption[6].

In Ref. [16], two prisms of the composite materials are used. The energy rays are measure and used to deduce the refraction index. The deviation between the two cases amounts to at least 40 percent. Reference [17] duplicates the experiment in Ref. [15] (Point V). The numerical simulation is not independent (Point IV). As pointed out in Ref. [8], a correct computer simulation starting with the microscopic Maxwell's equation would reproduce the experiments, but whether the observations can be explained in terms of a macroscopic Maxwell's equation with both negative permittivity and permeability remains an open question. Such an *ab initio* simulation is yet to come. In addition, as Pokrovsky and Efros pointed out[8], there are no doubts on the the results, but there is a need for solid confirmation of the data

interpretation.

3. Negative refraction by photonic crystals

The first proposals of negative refraction by photonic crystals were from Notomi[18] and Luo *et al.*[19], later experimentally tested by Cubukcu, *et al.* and Parimi *et al.*[20, 21, 22].

Most of photonic crystals studied so far are two dimensional arrays of dielectric cylinders. An advantage of this type of systems is that it can be solved *exactly* by the multiple scattering theory (MST), first formulated by Twersky[23], later reformulated for EM systems[24, 25], acoustic systems[26], and water waves[27]. Therefore we can have a rigorous examination or new predictions of this type of photonic crystals. We will adopt MST.

Imaging inside a photonic crystal. By the FDTD simulation, Notomi presented an image focusing inside a photonic crystal made of drilled holes in a uniform dielectric material. An focused imaging is presented in Fig. 13 of Ref. [18]. We have tried to reproduce the results, but failed, partially because the parameters have not been clearly given in Ref. [18]. In the simulation, however, we found that the snapshots of the imaging field depend significantly on the simulation time. When the stable stage has not been reached, some focused images like in [18] could be exhibited[28].

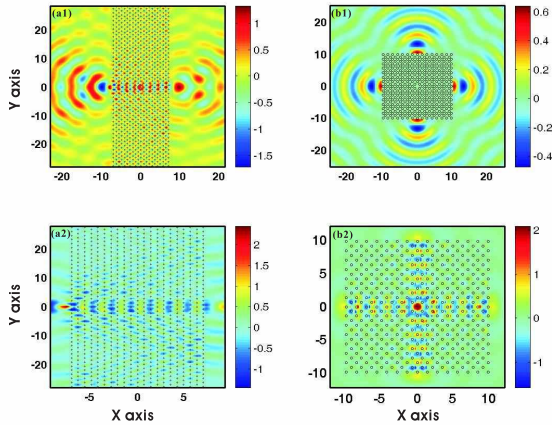


FIG. 5: (a1) and (a2): The imaging fields for a slab of photonic crystal structure. The slab measures as $10\sqrt{2} \times 40\sqrt{2}$. (b1) and (b2): The imaging fields for a transmitting source located inside a square array of cylinders. The square measures $14\sqrt{2} \times 14\sqrt{2}$.

Flat lens imaging - theoretical. The flat lens imaging illustrated by Fig. 3(b) was first studied in Ref. [19]. The systems considered are two types of photonic crystals. One is made of square arrays of dielectric cylinders in air. The other is made of square arrays drilled holes in a dielectric medium. Based upon the equal frequency band structure, the authors show there is a range of frequencies in which an all angle negative refraction can be archived in this scenario. We have done two verifications. First, we have reproduced nicely the observation of Ref. [19]. The results are shown in Fig. 1(c) of Ref. [29]. We indeed observe a focused image across the flat slab of photonic crystals.

If such an imaging property is due to negative refraction, we would then have two expectations (Point III): one is to find a focused image inside the slab as shown in Fig. 3(b); the other is to see the negative refraction effect in the scenario shown in Fig. 3(d).

For the first expectation, the inside focusing is hard to see in the arrays of Ref. [19], as their photonic crystal slab is too thin. We have thus enlarged the slab and plot both the amplitude fields and the intensity fields inside the slab. There is no focused image inside the slab. Instead, we observe that the energy transmission is guided through the slab, making the image. This has be further verified by putting an EM wave source inside a square shaped array of cylinders. The two sets of simulation results, taken from Ref. [29], are shown in Fig. 5. All the parameters and lattice arrangements are taken from Ref. [19]; all lengths are scaled by the lattice constant and the frequency is scaled by the radius of the cylinders. Here we see clearly the image focused on the other side of the slab in (a) and the four images made at the four exiting points are due to the guided or collimated wave transmission. It appears that there are favorable directions for waves to transport, as shown by (b2). We found that such collimated or guided transmission is caused by the presence of partial bandgaps. As can be seen from Fig. 3 of Ref. [19], the frequency chosen lies within a partial bandgap in which the waves are forbidden to transmit along the direction of $\Gamma - X$ in the square lattice. Due to this partial bandgap, waves seem to avoid the $\Gamma - X$ direction as much as possible, and thus appear to be collimated along the ΓM direction. The energy paths shown in Fig. 5 are actually along the $\Gamma - M$ direction.

We have also checked the second expectation. The results are presented in Ref. [30]. We found that the wave transmission always tends to be guided or collimated into the ΓM direction, independent of the incident angles or the orientation of the $\Gamma - M$ direction. This phenomenon cannot be classified as refraction either in the context of isotropy or anisotropy.

The results from Refs. [29, 30] led us to propose that partial bandgaps can serve as a guiding tunnel for the wave transport, allowing for new applications of photonic crystals[31]. Such a consideration has also been extended to acoustic systems[32].

The flat slab imaging discussed in Ref. [19] is not caused by negative refraction, rather it is caused by directed diffraction. The misinterpretation in Ref. [19] has also been pointed out by Li *et al.*[33]. At least, the explanation in Ref. [19] is unable to exclude other possible interpretations (Point I).

One may ask for the reason for the discrepancies between the present results and the analysis in Ref. [19]. Here we would like to share our thoughts. The problem with the conjectured flat slab imaging may lie in the approach to the energy flow inside the slab. The usual approach mainly relies on the curvatures of frequency bands to infer the energy flow.

As documented in Ref. [2], an energy velocity is defined as $\vec{v}_e = \frac{\frac{1}{V} \int \vec{J}_{\vec{K}} d^3r}{\frac{1}{V} \int U_{\vec{K}} d^3r}$, where $\vec{J}_{\vec{K}}$ and $U_{\vec{K}}$ are the energy flux and energy density of the eigenmodes, and the integration is performed in a unit cell, representing a spatially averaged value. It can be shown that thus defined energy velocity equals the group velocity obtained as $\vec{v}_g = \nabla_{\vec{K}} \omega(\vec{K})$. Therefore it is common to calculate the group velocity to infer the energy velocity and subsequently the energy flows or refraction of waves. Whether the net current flow through a unit cell really follows the direction of \vec{v}_e remains unclear. We note here that the average flux through a surface may be defined as $\langle \vec{J} \rangle = \frac{\hat{n}}{S} \int d\vec{S} \cdot \vec{J}$, where \hat{n} is the unit normal vector of the surface S . Clearly, the volume averaged current within a unit cell does not necessarily correspond to actual energy flows[34, 35]. General speaking, the periodically structured materials may not be regarded as an effective medium.

The curvature approach fails in the current partial bandgap case. For example, in the present case, the curvature method would lead to an almost 80 degree of energy deflection with reference to the ΓX direction when the incidence is at about 25 degree; The rigorous computational results show that the energy flow is nearly in the ΓM direction[30].

Flat lens imaging - experimental. Later, flat slab imaging has been experimentally observed by Parimi et al.[21]. The authors reported that an image can be formed cross a flat slab of photonic crystals made of dielectric cylinders. The appearance of the image is regarded as the occurrence of negative refraction and has thus been related to the negative refraction of LHMs. We found that their interpretation needs to be reconsidered. The focused image is accidental (Point III). The imaging pattern will completely change as the

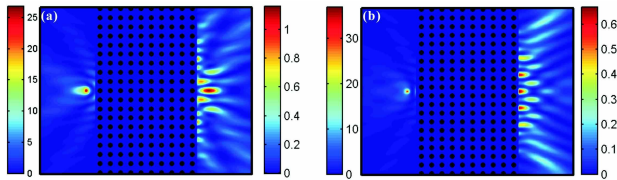


FIG. 6: The image of the intensity-fields for flat slabs. (a) and (b) refer to two arrangements. In (a), all the arrangements including the source, the field resolution, and the cylinders are identical to the experiment[21]. (b) is the same as (a) except that the source is move upward by a half lattice constant. The scales for the fields for the left and right sides are shown in the figure.

source is moved only by a half lattice constant. From the transmission calculation, we found that the image is due to the anisotropic scattering of the array of cylinders. Scattering by a group of materials certainly will yield bright-dark patterns, simply due to multiple scattering and wave interference, like in the X-ray imaging. Our rejected comments on the experimental observation are in Ref. [36]. Partial results are shown in Fig. 6. Fig. 6(a) reproduces remarkably well the experiment. In the case (b), the source is move upward by a half lattice constant. Here we see that the imaging field across the slab changes completely. Again, we do not criticize the experiment, but we think that the interpretation is ambiguous.

What is the difference between the flat slab imaging due to partial gaps and the imaging due to anisotropic scattering? The difference is that the imaging by partial gaps is a stable phenomenon, while the imaging due to scattering is unstable. The focusing phenomenon can be even seen by scattering by a single object under certain manipulation, as long as the scattering is anisotropic.

Plane wave through a slab of photonic crystals. Cubukcu *et al.* reported the measurement of a beam of EM waves incident on a flat slab of photonic crystals[20], referring to Fig. 3(d). The authors measured the energy ray directions. Then Equation (3) was used to obtain the refractive index. The author shows that at certain ranges of incident angles, the index is negative. As discussed earlier, Snell's law in the form of Eq. (3) is only applicable when the medium isotropic. However, the results in Ref. [20] indicate the opposite. This is clearly self-conflicting (Point II). In addition, we tried to reproduce the results in Fig. 2 of Ref. [20], but failed. We found the results similar to that in Ref. [30]. That is, independent of the incident angles, the waves are bent towards a particular direction, which is across the slab of photonic

crystals used in the experiment. A possible reason for the discrepancy may be due to the difference between the FDTD method used in [20] and the MST used in our simulation. As mentioned earlier, it takes a considerable long time for FDTD results to reach the stable stage. Without checking the stability, the FDTD results may be unreliable. Another possible cause is the finite size effect. In our simulation, we have excluded any thinkable numerical artifacts, with regards to the mode numbers, finite number of scatterers, boundary effects, and so on. We welcome any request for the numerical codes and our simulation procedures for independent and unbiased verification.

Prism scenario. Lastly but not least, we address the prism scenario. It is obvious that this scenario can only work when the medium is isotropic. Without first verifying that the medium is isotropic, the deduction of the refraction index is not reliable, as the waves may be already bent at the entering interface. As an example, we have simulated two prisms of the photonic crystals made of dielectric cylinders, given in Ref. [19, 29]. The results are presented in Ref. [37]. Here we show partial results.

We have plotted the transmitted intensity fields. The results are presented in Fig. 7. Here the incident waves are transmitted vertically from the bottom. The impinging frequency is $0.192 \cdot 2\pi c/a$ with a being the radius of the cylinder; the frequency has been scaled to be non-dimensional in the same way as in Ref. [19]. We note that at this frequency, the wave length is about five times of the lattice constant, nearly the same as that used in the experimental of [16]. The incidence is along the $[\cos 22.5^\circ, \sin 67.5^\circ]$ direction, that is, the incidence makes an equal angle of 22.5° with regard to the [10] and [11] directions of the square lattice; the reason why we choose this direction will become clear from later discussions. Moreover, on purpose, the intensity imaging is plotted for the fields inside and outside the prisms on separate graphs which have been scripted by ‘1’ and ‘2’ respectively.

First, we ignore the transmitted fields inside the prisms. From Fig. 7 (a1) and (b1) we are able to calculate the main paths of the transmitted intensities. The geometries of the transmission are indicated in the diagrams. The tilt angles of the prisms are denoted by ϕ , whereas the angles made by the outgoing intensities relative to the normals of the titled interfaces are represented by θ . According to the prescription outlined in Ref. [15, 16], once ϕ and θ are determined, Snell’s law is applied to determine the effective refractive index: $n \sin \phi = \sin \theta$. If the angle θ is towards the higher side of the prisms with reference to the normal, the angle is considered positive. Otherwise, it is regarded as negative. In

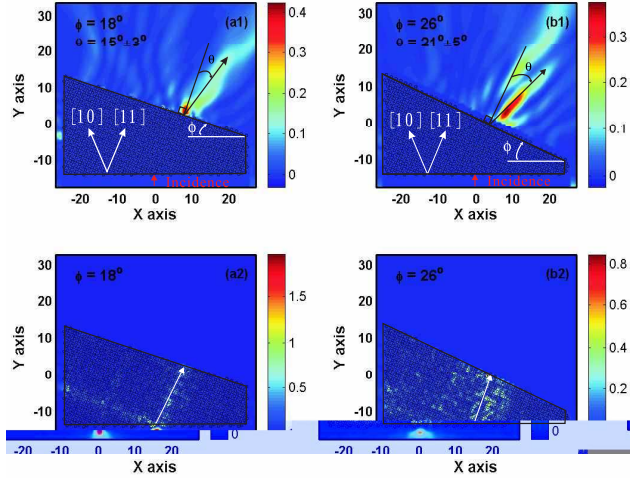


FIG. 7: The images of the transmitted intensity fields. Here, the intensities inside and outside the prism structures are plotted separately, so that the images can be clearly shown with proper scales. The geometric measurements can be inferred from the figure. The tilt angles for the two prisms are 18° and 26° respectively, and have been labelled in the figures. For cases (a1) and (b1), we observe the apparent ‘negative refraction’ at the angles of $15 \pm 3^\circ$ and $21 \pm 5^\circ$ respectively. When applying Snell’s law, these numbers give rise to the negative refraction indices of -0.84 ± 0.17 and -0.87 ± 0.21 for (a1) and (b1) separately. In (a2) and (b2), the intensity fields inside inside the prisms are plotted. Here we clearly see that the transmission has been bent. In the plots, [10] and [11] denote the axial directions of the square lattice of the cylinder arrays, i. e. $\Gamma - X$ and $\Gamma - M$.

light of these considerations, the apparent ‘negative refractions’, similar to the experimental observations[15, 16], indeed appear and are indicated by the black arrows in the graphs. After invoking Snell’s law, the negative refraction results are deduced. From (a1) and (b1), we obtain the negative refractive indices as -0.84 ± 0.17 and -0.87 ± 0.21 for the two prisms respectively. The inconsistency between the two values is less than 4%. The overall uncertainty in the present simulation is less than 24%. Therefore, a consistent negative refractive index may be claimed from the measurements shown in Fig. 7 (a1) and (b1). Furthermore, we found that with certain adjustments such as rotating the arrays or varying the filling factor, the index obtained by Snell’s law can be close to the perfect -1.

Now we take into consideration the wave propagation inside the prisms. As shown by Fig. 1 (a2) and (b2), the transmission inside the prisms has been bent at the incidence interfaces, referring to the two white arrowed lines in (a2) and (b2). If Snell’s law were valid

at the outgoing interfaces, it should also be applicable at the incident boundaries. Then with the zero incidence angle and a finite refraction angle, Snell's law would lead to an infinite refractive index for the surrounding medium. Besides, when taking into account the bending inside the prisms, the incident angle at the outgoing or the tilted surface is not θ any more. Therefore the index value obtained above is incorrect. Our opinion is that we need to differentiate the refraction from diffraction. The phenomenon shown in Fig. 7 should be attributed as occurrence of diffraction effects.

The measurements in Ref. [22] have adopted the prism scenario. The photonic crystals are made of cylinders, and made into a prism. Then the energy rays are measured from the incident and outgoing waves, to obtain refractive index by Snell's law. Again, the authors have not verified the isotropy. How waves propagate inside the prism has not been probed. According to the results in Fig. 7, the experiment is not sufficient to identify the LHM behavior of the photonic crystals.

V. SUMMARY

In summary, we here discuss some fundamental questions about LHMs and the associated negative refraction. It is pointed out that the current experimental evidence is not sufficient to conclude that LHMs have been formulated. Some concerns and considerations are presented to support the analysis. The key is to carry out a comprehensive experimental exploration, with regard to all aspects of LHMs and negative refraction, including such issues as phase information, isotropy, and effectiveness etc. The phase information will be essential to differentiate the negative refraction from the quasi-negative refraction.

Before talking about either the positive or negative refraction, a prior task is to find out whether it is the refraction phenomenon that occurs when waves pass across the interface between two media. It is insufficient to just look at the deflection of the energy intensity path to discern the refraction phenomenon. Otherwise, confusions can accumulate[38]. In sonic crystals, it can be shown that negative refraction can never occur in the sense of LHMs. We think that the report in Ref. [38] misleading. Structured materials can certainly manipulate wave propagation in various ways. However exciting, it is crucial to confirm first the manipulation is due to the refraction mechanism before classifying as negative or positive refraction, and claiming the finding or fabricating of LHMs.

The essence of the present work is not to discard the previous efforts, rather it is hoped to help clarify some of possible uncertainties in the previous data interpretation or theoretical analysis. Only through the detailed examination, debate or introspection, the understanding of the problem can be deepened. Given the situation that the issue of LHMs has ignited such a huge response, a careful reconsideration of the problem is always something important.

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